

The Factor Tau in the Black-Litterman Model

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Abstract

This paper considers the factor tau (τ) in the Black-Litterman model. It is one of the more confusing aspects, as authors provide contradictory information regarding its use and calibration. We will consider the origin of the mixed-estimation model used in the Black-Litterman Model so that we can develop a richer understanding of τ and when it should be used, and if used how it should be calibrated. We will show that most practitioners can benefit from using an alternative reference model explicitly without τ . For those who want to use τ we will provide a discussion on how it should be calibrated.

Introduction

The Black-Litterman Model is a model for estimating asset returns. It has two main features, the use of an informative prior derived from the CAPM equilibrium, and a mixing model that allows the investor to specify views on any linear combination of the assets. The mixed-estimation model was originally developed by Theil and Goldberger (1961). The mixing model allows absolute and relative views, and views may be on any combination of the assets. A summary of the literature and more details on the Black-Litterman Model can be found in Walters (2008).

This paper will focus on the role of the factor tau (τ). τ is used to scale the investors uncertainty in their prior estimate of the returns. There are several different approaches to calibrating it, or even including it described in the literature. Just to illustrate the difference of opinion, we will look at comments from three authors. He and Litterman (1999) state they set $\tau = 0.05$. Satchell and Scowcroft (2000) state many people use a value of τ around 1. Meucci (2010) proposes a formulation of the Black-Litterman model without τ .

We introduce a concept called the Reference Model to explain the differences between the various authors. We will start by presenting the Original Reference Model as derived from Theil's mixed estimation approach in Black and Litterman (1991). This discussion will be enriched with information from He and Litterman (1999). Next we will present the Alternative Reference Model as proposed by Meucci (2010) which estimates the returns without τ . Finally, we will provide recommendations on whether to include τ , and if so how to calibrate τ .

The Black-Litterman Model

This section will provide an overview of the Black-Litterman Model. The reader can consult one of the references for more details.

We can view the process of using the Black-Litterman Model as having three distinct steps. The first step is the calculation of the informative prior estimate of returns. The prior is derived from the CAPM equilibrium portfolio using the following formula which is the closed form solution to unconstrained Mean-Variance optimization.

$$(1) \quad \Pi = \delta \Sigma w$$

Formula (1) is the relationship which we call reverse optimization. Given the risk aversion of the market, δ , the covariance of returns, Σ , and the equilibrium weights, w , we can back out the expected equilibrium returns. Π will be our prior estimate of the mean return.

The second step is the specification of the investor's views. Here the investor formulates estimated returns and uncertainties for one or more view portfolios, or linear combinations of the assets.

The third step is the mixed estimation process used to blend the prior estimates of returns with the views to create the posterior estimates of the returns along with estimates of the uncertainty of the estimates. We can arrive at the same formulation for the blending process using the standard case of an unknown mean and a known variance from Bayesian theory.

We start with with the assumption of normally distributed expected returns. The goal of the Black-Litterman model is to estimate the parameters of this distribution.

$$(2) \quad E(r) \sim N(\mu, \Sigma_\mu)$$

μ Mean return

Σ_μ Covariance matrix for the distribution of returns about the mean

There are two widely used models for the blending process. We call these Reference Models. Each Reference Model contains different assumptions about what parameters to use in order to model formula (2).

The Original Reference Model

This section reviews the Original Reference Model as defined in the papers, Black and Litterman (1991), and He and Litterman (1999).

First we introduce the simple linear model from Theil and Goldberger (1961) for the estimated return.

$$(3) \quad \pi = \mu + \epsilon$$

The core of the Original Reference Model is the concept that the investor is uncertain of their estimate π , for μ , the mean return, as shown in formula (3). “*The mean is an unobservable random variable*”, Black and Litterman (1991). With μ as a stochastic variable, then ϵ the residual has a probability distribution. We assume ϵ is normally distributed with mean 0 and variance Σ_π . Thus, we can state that Σ_π is the variance of the investor's estimate about the mean return μ . Theil (1970) uses the phrase “sampling variance” for Σ_π . Standard error is another name for the square root of sampling variance.

This leads to the following expression for the distribution of the estimated return about the mean return.

$$(4) \quad \pi \sim N(\mu, \Sigma_\pi)$$

Formula (4) shows the distribution of the estimate of the mean return about the unknown mean return. This is analogous to the situation where we perform the following process. Each of n times we draw m realizations from the population of returns and compute the mean of each sample. Then we view the distribution of the sample means around the population mean. π is the mean of the sample means, and Σ_π is the sampling variance of the distribution of sample means about the population mean. We expect in the limit as n approaches infinity that π approaches μ and Σ_π approaches 0.

In the case of normally distributed samples about the mean, the sampling variance is

$$(5) \quad \Sigma_{\pi} = \frac{\Sigma_{\mu}}{n}$$

We assert for simplicity that Σ_{π} and Σ_{μ} are independent and uncorrelated, then Σ_r , the variance of the distribution of returns about the estimated mean, π , is given by formula (6).

$$(6) \quad \Sigma_r = \Sigma_{\mu} + \Sigma_{\pi}$$

Given π as our estimate of μ , then Σ_r is essentially our estimate for Σ_{μ} .

We can check the reference model at the boundary conditions to ensure that it is correct. In the absence of estimation error, e.g. $\varepsilon \equiv 0$, then $\pi = \mu$ and $\Sigma_r = \Sigma_{\mu}$. As our estimate gets worse, e.g. Σ_{π} increases, then Σ_r increases as well. This behavior is consistent with our earlier assertion that our posterior estimate of the mean is more precise than either the views or the prior. In addition it is also consistent with the idea that estimates of the variance of a distribution of a financial time series about an estimated mean, can at best approach a lower limit which is the variance of the distribution about the population mean. It cannot go below that value.

To further simplify the model we can assert that Σ_{π} is proportional to Σ_{μ} where the constant of proportionality is known as τ . This assertion is useful since we usually estimate Σ_{μ} rather than Σ_{π} .

$$(7) \quad \Sigma_{\pi} = \tau \Sigma_{\mu}$$

If we combine formulas (5) and (7), we can relate τ and n .

$$\Sigma_{\pi} = \tau \Sigma_{\mu} = \frac{\Sigma_{\mu}}{n}$$

$$(8) \quad \tau = \frac{1}{n}$$

If we used a statistical process to formulate our prior estimate, then we would have a clear method for calibrating it based on this relationship.

Now we can introduce our expression for the Original Reference Model for the estimated distribution of expected returns.

$$(9) \quad E(r) \sim N(\pi, (\Sigma + \Sigma_{\pi})), \quad \pi \sim N(\mu, \Sigma_{\pi})$$

Formula (9) represents the complete Original Reference Model which corresponds to our goal as defined in formula (2). This reference model matches up with formulas 8, 9 and 10 in He and Litterman (1999).

We will not show the derivation here as it can be found in several of the references, but the standard expression for the Black-Litterman posterior estimated mean and sampling variance is:

$$(10) \quad \hat{\Pi} \sim N([\tau \Sigma]^{-1} \Pi + P^T \Omega^{-1} Q) [\tau \Sigma]^{-1} + P^T \Omega^{-1} P]^{-1}, [(\tau \Sigma)^{-1} + P^T \Omega^{-1} P]^{-1})$$

$\hat{\Pi}$ Posterior estimate of the mean returns

τ A scaling factor for the uncertainty of the estimated equilibrium mean return

Σ Covariance matrix for the distribution of returns about the actual mean

Π Estimated equilibrium mean returns (prior estimate)

P View selection matrix

Ω Covariance of the estimated view mean returns about the actual view mean returns

Q Estimated mean returns for the views

$\tau\Sigma$ is the sampling variance or uncertainty in the prior mean return estimate. Ω is a similar term for the uncertainty of the estimated returns of the views. In this reference model, Ω is not the variance of the distribution of returns of the views.

The discussion of formula (10) is easier in terms of the inverse of the covariance matrix, a term known as precision in the Bayesian literature. We can summarize the posterior estimated mean in formula (10) as the precision weighted average of the prior estimate and the view estimates. The posterior precision is the sum of the prior and view precisions. Both these formulations match our intuition as we expect the precision of our posterior estimate is more than the precision of either the prior or the views. Second, the mixed estimation process should make use of the precision of the estimates in the weighting of the mixing, e.g. an imprecise estimate should have less impact on the posterior than a precise estimate.

With a small modification to the covariance term we can rewrite (10) using (5) to be an expression for the Black-Litterman posterior estimate of the mean and covariance of returns around the mean.

$$(11) \quad E(r) \sim N([\tau\Sigma]^{-1}\Pi + P^T\Omega^{-1}Q)[[\tau\Sigma]^{-1} + P^T\Omega^{-1}P]^{-1}, (\Sigma + [\tau\Sigma]^{-1} + P^T\Omega^{-1}P)^{-1})$$

In the Original Reference Model, the updated sampling variance of the mean estimate will be lower than either the prior or conditional sampling variance of the mean estimate, indicating that the addition of more information will reduce the uncertainty of the model. In Bayesian terms, the estimation process increases the precision of the estimate.

The variance of the returns from formula (11) will never be less than the known variance of returns about the mean. This matches our intuition as adding more information should reduce the uncertainty (increase the precision) of the estimates, but cannot reduce the covariance beyond that limit. Given that there is some uncertainty in variance about the mean, then formula (11) provides a better estimator of the variance of returns about our estimated mean than the known variance about the mean.

The Alternative Reference Model

This section will discuss the Alternative Reference Model described in Meucci (2010). This model is commonly used in the literature, though usually not explicitly. While it has been essentially described by other authors, Meucci explicitly described its features. We will further assert that any author who suggests τ with a scale of 1, or does not use an updated posterior variance is using this reference model implicitly.

We start with the expression for the distribution of the estimated returns about the population mean shown in formula (2).

$$E(r) \sim N(\mu, \Sigma)$$

In the Alternative Reference Model, the estimated returns are normally distributed around the mean μ with variance Σ_μ . However the linear model is different from the Black-Litterman reference model. Remembering the linear model from the previous section.

$$\pi = \mu + \epsilon$$

π is our estimate of μ , but μ is no longer a random variable. In the Alternative Reference Model, ϵ is an

unknown constant so $\Sigma_\pi \equiv 0$.

The Alternative Reference Model is commonly described as having a $\tau = 1$ because the prior variance used in the mixing process is $\Sigma_\pi = \Sigma_\mu$. More precisely we have eliminated τ as a factor. If we look back to the Black-Litterman reference model, $\Sigma_\pi = \tau\Sigma$, so if $\Sigma_\pi = 0$ then $\tau = 0$. It cannot both be 0 and 1 at the same time.

Our reformulated formula for the posterior distribution of estimated returns, matching previous formula (11) is now

$$(12) \quad E(r) \sim N([\Sigma^{-1}\Pi + P^T\Omega^{-1}Q][\Sigma^{-1} + P^T\Omega^{-1}P]^{-1}, \Sigma)$$

We have removed τ from the formula for the posterior estimate of the mean and we have changed the posterior covariance matrix to be the known covariance matrix about the mean. Note in this model the sampling variance is ignored and we ignore the posterior precision.

The Alternative Reference Model does not use a sampling variance, but instead uses the covariance of the distribution of returns around the mean for the prior covariance. This means the posterior covariance from the mixing model is not used, instead the prior estimate of the covariance matrix is used instead.

We can check this methodology as follows. If we consider the case where Ω and Σ are approximately equal, then given our mixing model we would expect the posterior precision to be twice the prior precision. This is not intuitively satisfactory. If we consider a financial time series with a standard deviation of 15%, we don't expect the standard deviation to be reduced to 10% during the process of updating the estimate of the mean. This conflicts with our earlier statements that higher precision in our estimate of the mean return should not cause the covariance of the return distribution to shrink by the same amount. Thus, we don't update the posterior covariance in the alternative model, and it remains the prior covariance Σ_μ .

The terms used to control the weights in the blending are now the variance of returns about the mean for the prior, Σ_μ , and the views, Ω . The investor calibrates Ω in order to control the weights on the views versus the weights on the prior estimates. This provides enough degrees of freedom to achieve any ratio of the prior to the views, so removing τ does not negatively impact the estimating process. In fact if we take formula (11) and perform a little algebra, multiply numerator and denominator by τ and replace the posterior covariance with the prior covariance, we arrive at

$$E(r) \sim N([\Sigma^{-1}\Pi + P^T\tau\Omega^{-1}Q][\Sigma^{-1} + P^T\tau\Omega^{-1}P]^{-1}, \Sigma)$$

Thus our statement about no loss of degrees of freedom is true. In the updated estimate above we see that we've replaced Ω with Ω/τ in the expression for the mean, but otherwise it matches formula (12). This means we can just scale our Original Reference Model Ω by $1/\tau$ and still get the same results for the mean using the Alternative Reference Model.

The method proposed in Idzorek (2005) calibrates Ω given an investor specified confidence level for each view. This makes the Black-Litterman much more accessible with no loss of features. When using the Idzorek method the investor specifies their confidence as a percentage which represents the fraction of the change in returns between 0% confidence and 100% confidence. Though his paper includes τ , he does not use an updated posterior covariance. His calibration process for Ω scales it vs $\tau\Sigma$, but can just as easily work vs Σ alone. Thus, when using the Idzorek method our investor should use the Alternative Reference Model.

Choosing a Reference Model

Now that the two reference models have been described we can provide some guidance on using one model or the other. The most common reference model in the literature is the Alternative Reference Model. Meucci (2010), Idzorek (2005) and many other authors use this model. We can see this by examining which authors update the posterior precision, or use the covariance portion of formula (11), and which authors just use the prior covariance, use formula (12). If they do not update the posterior precision and covariance, then they are using the Alternative Reference Model. In this case the value selected for τ turns out to be unimportant and can be safely forgotten.

The primary reason to use the Original Reference Model would be to pick up the additional information from the model via the updated posterior covariance matrix. This additional information from the model comes at the cost of needing to determine the additional factor τ . Investors willing to accept the simpler Alternative Reference Model can avoid the need to consider τ .

Worked Example with Both Reference Models

When we use the Original Reference Model, the posterior covariance may be smaller than the prior covariance if the views improve the precision of the estimate. Absolute views generally make larger improvements in the precision of the estimate. Relative views make weak or no improvements in the precision of the estimate. Intuitively this makes sense as relative views do not provide an improved estimate of the mean, just extra information on the relationship between the estimates. We can measure the precision of the estimates by summing the unconstrained weights. We can compute an effective posterior measure of uncertainty/precision as shown below

$$(13) \quad \eta = \frac{(1 - \sum_1^n w_i)}{\sum_1^n w_i}$$

The value η in formula (13) can be compared to τ and can be used to measure the uncertainty in the prior or posterior estimates of the mean. When viewing the prior estimates, $\eta = \tau$. We can compare η between the prior and the posterior to determine the relative improvement in the precision of the estimates..

One of the interestingly artifacts of the Black-Litterman model is that while the estimated return of an asset without views can change, the unconstrained weight of the asset in the portfolio changes very little if at all. This is true regardless of which reference model we use. Under the Original Reference model, our investor with less than 100% confidence in their prior estimates is not 100% invested, but is only invested in the fraction $1/(1 + \tau)$. This is because of formula (6) which shows the prior dispersion of realized returns about the estimated mean is $(1 + \tau)\Sigma_\mu$. The asset weights in an unconstrained portfolio based only on the prior will be $w_{eq}/(1 + \tau)$. Because the posterior precision of the estimated mean will be equal or higher, the investor will invest an equal or larger fraction of their wealth in the portfolio and the asset allocation will experience some change solely because of this change. In the Alternative Reference Model the unconstrained weights will generally sum to 1, and we do not consider precision of the posterior estimates. Of course, since most portfolio optimization is constrained, the final asset weights will most likely change even when an investor has no view on a specific asset.

Now we will work an example using both the Original Reference model and the Alternative Reference model. The details of the example can be found in Appendix A. We examine two scenarios

- Relative Views – Investor has two relative views
- Absolute View – Investor adds an absolute view on Germany with return = return from the first scenario.

This construction should allow scenario two to illustrate the difference in the unconstrained weights caused solely by the updated posterior covariance matrix.

Table 1 – Black-Litterman Reference Model Results

Asset	Equilibrium Weights	BL Ref Model Prior Weights	BL Ref Model Relative Views	BL Ref Model Absolute View
Brazil	3.39%	3.23%	11.89%	11.85%
China	6.32%	6.02%	22.16%	22.10%
France	8.23%	7.84%	21.60%	21.41%
Germany	9.96%	9.49%	26.14%	27.54%
Japan	13.56%	12.91%	12.91%	12.91%
UK	11.81%	11.25%	-19.18%	-18.74%
US	46.73%	44.50%	19.70%	19.80%
Sum	100.00%	95.24%	95.24%	96.88%
η		0.050	0.050	0.032

Table 1 shows that when using the Original Reference Model the unconstrained weights of the assets without views (Japan) do not change from the prior unconstrained portfolio. Note that because of the investors confidence in the prior is less than 100%, the prior asset weights differ from the equilibrium by a factor of $1/(1 + \tau)$. We can also see that an absolute view which only changes the posterior covariance will cause changes to the unconstrained asset allocation, but only to the assets with views. In this case an absolute view of moderate confidence caused an additional 140bps shift in the unconstrained weights of the asset in the view. Other assets with views had changes in their allocation from 4 bps for an asset in an unrelated view, to 19bps and 44bps for assets coupled to Germany through a relative view.

Table 2 – Alternative Reference Model Results

Assets	Equilibrium Weights	Alt Reg Model Prior Weights	Alt Ref Model Relative Views	Alt Ref Model Absolute View
Brazil	3.39%	3.39%	12.43%	12.43%
China	6.32%	6.32%	23.17%	23.17%
France	8.23%	8.23%	22.25%	22.25%
Germany	9.96%	9.96%	26.93%	26.93%
Japan	13.56%	13.56%	13.56%	13.56%
UK	11.81%	11.81%	-19.18%	-19.18%

US	46.73%	46.73%	20.84%	20.84%
Sum	100.00%	100.00%	100.00%	100.00%

Table 2 shows that in the Alternative Reference Model the prior weights match the equilibrium weights as uncertainty in the estimated covariance is not a factor. It also shows that the posterior unconstrained weight of an asset included in no views does not change from the equilibrium weight. Further it shows no impact from the absolute view on Germany, the expected return is already taken into account by the results from scenario 1.

With both reference models the unconstrained weights for an asset with no views, e.g. Japan, do not change. This matches our assertion earlier that this is a general property of the Black-Litterman model.

Clearly the Original Reference Model leads to larger movements from the equilibrium. This is because we are simultaneously updating our estimate of the mean, and increasing the precision of that estimate. This causes the asset allocation to tilt subtly more toward the assets included in the views with higher precision. The impact of the updated posterior covariance is a second order effect, as we can see from Table 1 it is less than 10% of the total change from the prior unconstrained weight.

Setting τ in the Black-Litterman Model

Now we will consider how to select a specific value of τ in more detail. A brief survey of the literature will be helpful. Given the previous discussion, we will focus on authors who use the Original Reference Model in this section.

From Black and Litterman (1991)

Because the uncertainty in the mean is much smaller than the uncertainty in the return itself, τ will be close to zero. The equilibrium risk premiums together with tS determine the equilibrium distribution for expected excess returns. We assume this information is known to all; it is not a function of the circumstances of any individual investor.

He and Litterman (1999) propose considering τ as the ratio of the sampling variance to the distribution variance, and thus it is $1/t$. They use a value of τ of 0.05 which they describe as

“...corresponds to using 20 years of data to estimate the CAPM equilibrium returns.”

As described previously, τ is the constant of proportionality between Σ_π and Σ . We will examine three ways in which we might select the value for τ . It is important to remember that τ is a measure of the investor's confidence in the prior estimates, and as such it is largely a subjective factor.

We will consider three methods to select a value for τ

- Estimate τ from the standard error of the equilibrium covariance matrix
- Use confidence intervals
- Examine the investor's uncertainty as expressed in their prior portfolio

First, we will approach the problem from the point of view of He and Litterman (1999). If we were using regression techniques to find π using formula (3), then Σ_π would be the sampling variance or square of the standard error of the regression. Formula (14) is the expression for the standard error where the residual is normally distributed which is assumed in the model.

$$(14) \quad Std\ Err = \frac{\sigma}{\sqrt{n}}$$

In the Black-Litterman model we do not use a regression approach to find the prior estimate of the mean return, we solve for it using equilibrium techniques that have no clear standard error term. We do however generate the prior covariance matrix using a sample of returns, and we have a consistent n to use with our Σ_{μ} which we could use to estimate a standard error for the estimate of the mean.

Formula (8) shows the basic relationship between τ and n from our covariance matrix calculations. Because we are using non-statistical methods to estimate the mean return, this is not a quantitative answer as to what value we should use for τ , it is just one way to provide some intuition around the scale of τ .

A second approach to establishing a reasonable value for τ is to use confidence intervals. This has a more direct connection with the model and our estimate of the model. Formula (4) illustrates the distribution of the estimate of the mean, about the mean return. From this distribution we can assert a confidence interval for our estimate using basic probability.

A plausible scenario we might encounter would be yearly equity like returns with $\mu = 8\%$ and $\sigma = 15\%$. Table 3 below shows the 95% and 99% confidence intervals for this scenario and various values of τ .

Table 3

τ	95% Confidence	99% Confidence
0.0167	$\mu \in (4.13\%, 11.87\%)$	$\mu \in (2.19\%, 13.81\%)$
0.0250	$\mu \in (3.26\%, 12.74\%)$	$\mu \in (0.88\%, 15.12\%)$
0.0500	$\mu \in (1.30\%, 14.71\%)$	$\mu \in (-2.06\%, 18.06\%)$
0.2000	$\mu \in (-5.42\%, 21.42\%)$	$\mu \in (-12.12\%, 28.13\%)$
1.0000	$\mu \in (-22.00\%, 38.00\%)$	$\mu \in (-37.00\%, 53.00\%)$

We can see setting τ too high makes a very weak statement for our prior estimate of the mean. For example where we select $\tau = 0.20$ then our estimate at the 99% confidence level is about $8\% \pm 20\%$ which is not a very precise estimate for the mean of a distribution.

Third, we can consider τ from the point of view of a Bayesian investor. Given the uncertainty in the estimates, our Bayesian investor begins the process not fully invested, in fact the fraction of their wealth invested is $1/(1+\tau)$. A value of $\tau = 1$ would lead to the investor being only 50% invested based on the prior estimates. A value of $\tau = 0.25$ would lead to the investor being 80% invested based on the prior estimates. The value η which we defined in formula (13) is the measure of wealth invested given the posterior estimates. A plausible example of this case would be a prior asset allocation of 90-95% which results in a value of τ of between .053 and .11.

Summary

We have seen the derivation of the Black-Litterman model from first principles and what the value τ represents. Two reference models which are in common use were presented. The Original Reference Model includes second order effects from an updated covariance matrix, but requires the investor to

deal with an additional factor, τ . The Alternative reference model is simpler, ignoring the second order impact of the posterior covariance matrix in exchange for disposing of τ all together. Most of the Black-Litterman literature makes use of the Alternative Reference model explicitly or implicitly, and investors are well served by using the Alternative Reference Model.

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Appendix A

Appendix A contains the specification of the example scenario. Note that this is essentially random data and any correspondence with a real state of the market is purely coincidental. It is for example purposes only.

MATLAB code implementing the example is available at <http://www.blacklitterman.org>.

We have the following correlation matrix, equilibrium weights and standard deviations of excess returns.

Correlation	Brazil	China	France	Germany	Japan	UK	US
Brazil	1.0000	0.4118	0.2830	0.4192	0.4227	0.2771	0.4022
China	0.4118	1.0000	0.6994	0.7044	0.3220	0.7203	0.6665
France	0.2830	0.6994	1.0000	0.7231	0.2868	0.7124	0.5032
Germany	0.4192	0.7044	0.7231	1.0000	0.2933	0.7126	0.5164
Japan	0.4227	0.3220	0.2868	0.2933	1.0000	0.3400	0.2607
UK	0.2771	0.7203	0.7124	0.7126	0.3400	1.0000	0.6011
US	0.4022	0.6665	0.5032	0.5164	0.2607	0.6011	1.0000
Weight	3.39%	6.32%	8.23%	9.96%	13.56%	11.81%	46.73%
Std Dev	24.20%	18.50%	17.00%	16.81%	19.61%	15.40%	19.11%

We use $\delta = 2.5$ for the risk aversion of the market and $\tau = 0.05$.

The first view is that emerging markets returns will exceed US returns by 2% with uncertainty equal to the diagonal of $P\tau\Sigma P'$ (precision of the view = precision of the prior).

The second view is that returns from France and Germany will exceed UK returns by 2% with the same precision as in the first view.

The third view is that German returns will be 5.158% (posterior when views 1 & 2 are applied) with precision as in the first and second view.